Lyapunov exponents and oceanic fronts

Francesco Toselli, Francesco d'Ovidio, Marina Lévy, Francesco Nencioli, and Olivier Titaud

Sorbonne Universités (UPMC, Univ Paris 06), LOCEAN Laboratory
4 place Jussieu, 75005 Paris, France

Abstract. Lyapunov exponents and Lyapunov vectors are precious tools to study dynamical systems: they provide a mathematical framework characterizing sensitive dependence on initial conditions, as well as the stretching and the contraction occurring along a trajectory. Their extension to finite size and finite time calculation has been shown to lead to the location of Coherent Lagrangian Structures, which correspond in geophysical flows to frontal regions. In this case, the Lyapunov exponent and the Lyapunov vector provide respectively the cross front gradient amplification and the front orientation. Here we present global maps of Lyapunov exponents/vectors computed from satellite-derived surface currents of the oceans and we quantify their capability of predicting fronts by comparing with Sea Surface Temperature images. We find that in high energetic regions like boundary currents, large relative separations are achieved in short times (few days) and Lyapunov vector mostly align with the direction of jets; in contrast, in lower energetic regions (like the boundaries of subtropical gyres) the Lyapunov calculation allows to predict tracer lobes and filaments generated by the chaotic advection occurring here. These results may be useful for a global calibration and validation of the Lagrangian technique for multidisciplinary oceanographic applications like co-localization of marine animal behaviours to frontal systems and adaptive strategies for biogeochemical field studies.

The ocean is a turbulent system where its physical and biogeochemical tracers (like heat, salinity, phytoplankton) present strong inhomogeneities that are structured over a large range of spatiotemporal scales by features like vortices (eddies) and fronts. Several methods have been proposed to analyze the surface currents and track the physical features that constrain tracer distributions through the horizontal transport. In particular, Lagrangian methods allow to mimic the transport dynamics by creating synthetic particle trajectories which are obtained by integrating the velocity field and then analyzed.

One powerful diagnostic which has been used to identify frontal structures, i.e. lines where discontinuities or strong gradients are expected to occur in the ocean, is the calculation of the local Lyapunov exponent. In general the Lyapunov exponents are used in a dynamical system approach in order to detect chaotic behaviour for an invariant system by measuring the growth of the perturbations occurring along particle trajectories. For geophysical systems, the calculation of the Lyapunov exponent is usually performed at finite time and finite space.

When computed in this way along backward trajectories, this calculation has been shown to provide exponent maxima (ridges) along frontal structures. Intuitively, this is due to the fact that one common mechanism for producing a front in the ocean is by confluence of water masses originally located far apart, so that the trajectories of nearby particles in a frontal region diverge fast when observed evolving backward in time [1–3, 7]. Correspondingly, the Lyapunov vector associated to the leading exponent contains information on the front orientation [6]. Many studies have been conducted in the past [4, 5, 8].
Here we explore the reliability of the finite size Lyapunov technique applied to satellite-derived ocean currents, in estimating front orientation. In order to achieve this we compute systematically Lyapunov exponent and vectors and we compare to the direction of sea surface temperature images observed from microwave satellites. We study this comparison by focusing to the North Atlantic

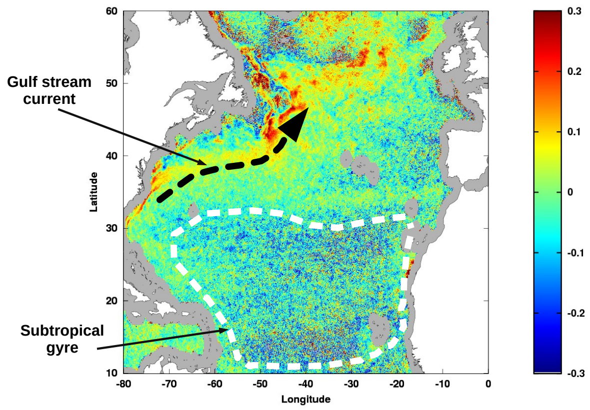

Fig. 1: Assessment of the capabilities of the Lyapunov analysis in predicting the orientation of SST gradients in the North Atlantic. Regions in blue (in particular the subtropical gyre) correspond to areas where the Lyapunov analysis predicts SST gradient orientations better than current direction. Red regions (in particular higher energetic features like the Gulf Stream) are areas where current direction performs better. See text for details. Altimetry data were provided by AVISO/CLS with support from CNES. The map shows 2003-2007 mean (degrees) computed in frontal regions (defined as Lyapunov exponents larger than 0.1 day^{-1}).

basin where we analyze and calculate the angles between Lyapunov vectors, SST gradient direction and surface velocity fields vectors, in order to measure how much better Lyapunov predicts fronts in respect to the direction of the flow. Firstly we compute two angular difference tests: one between velocity fields and SST gradient and one between Lyapunov vectors and SST gradient. After that we calculate the difference between these two tests, showed in Figure 1, in order to assess the skill of the Lyapunov analysis for predicting SST gradient orienta-

tion in respect to velocity directions.
Figure 1 presents a preliminary results where we can see differences between zones of high kinetic energy, like the Gulf Stream Current, where Lyapunov vectors mostly align with the direction of currents. On the other hand in lower energetic regions, like the subtropical gyre, Lyapunov vectors appear as a better predictor of SST gradient than current direction.

Scientific Validation

This paper has been unanimously validated in a collaborative review mode with the following reviewers:

– Rens Meerhof
– Laure Resplandy, LOCEAN-Paris
– Maria Carmen Lemos, School of Natural Resources and Environment-University of Michigan

References

1. Edward R. Abraham and Melissa M. Bowen. Chaotic stirring by a mesoscale surface-ocean flow. *Chaos*, 12(2):373–381, 2002.
2. F. J. Beron-Vera, M. J. Olascoaga, and G. J. Goni. Oceanic mesoscale eddies as revealed by lagrangian coherent structures. *Geophysical Research Letters*, 35(12):n/a–n/a, 2008. L12603.
3. G. Boffetta, G. Lacorata, G. Redaelli, and A. Vulpiani. Detecting barriers to transport: a review of different techniques. *Physica D: Nonlinear Phenomena*, 159(12):58 – 70, 2001.
4. Francesco d'Ovidio, Silvia De Monte, Séverine Alvain, Yves Dandonneau, and Marina Lévy. Fluid dynamical niches of phytoplankton types. *Proceedings of the National Academy of Sciences*, 107(43):18366–18370, 2010.
5. Francesco d'Ovidio, Silvia De Monte, Alice Della Penna, Cedric Cotté, and Christophe Guinet. Ecological implications of eddy retention in the open ocean: a lagrangian approach. *Journal of Physics A: Mathematical and Theoretical*, 46(25):254023, 2013.
6. Y. Lehahn, F. d'Ovidio, M. Lévy, and E. Heifetz. Stirring of the northeast atlantic spring bloom: A lagrangian analysis based on multisatellite data. *Journal of Geophysical Research: Oceans*, 112(C8):n/a–n/a, 2007. C08005.
7. Shawn C. Shadden, Francois Lekien, and Jerrold E. Marsden. Definition and properties of lagrangian coherent structures from finite-time lyapunov exponents in two-dimensional aperiodic flows. *Physica D: Nonlinear Phenomena*, 212(34):271 – 304, 2005.
8. Darryn W. Waugh and Edward R. Abraham. Stirring in the global surface ocean. *Geophysical Research Letters*, 35(20):n/a–n/a, 2008. L20605.